Making Direction a First-Class Citizen of Tobler's First Law of Geography

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Tobler's First Law of Geography (TFL)

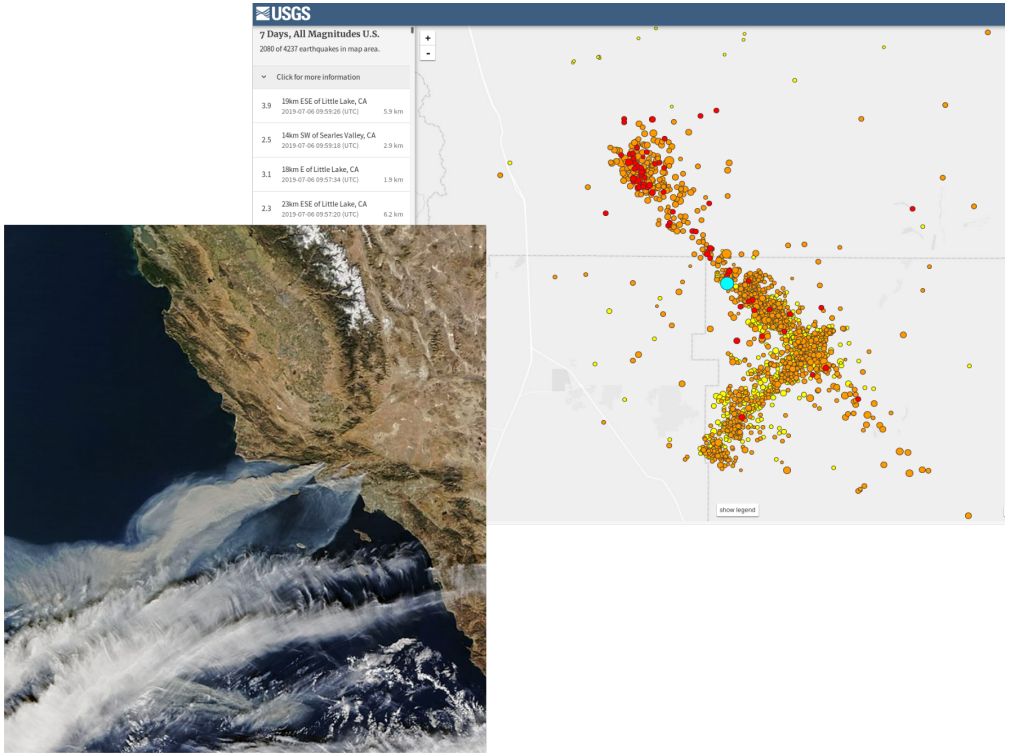
Waldo Tobler (1930 - 2018)

"Everything is related to everything else, but **near** *things are more related than* **distant** *things"* (Tobler, 1970)

The Implicit Assumption behind TFL

- Spatial relations
  - distance
  - **direction**
- Isotropicity
  - the **invariance** to rotations
  - $f(x^{'}) = f(Rx) = f(x)$ where R is the rotation matrix
  - **directional variation** is negligible

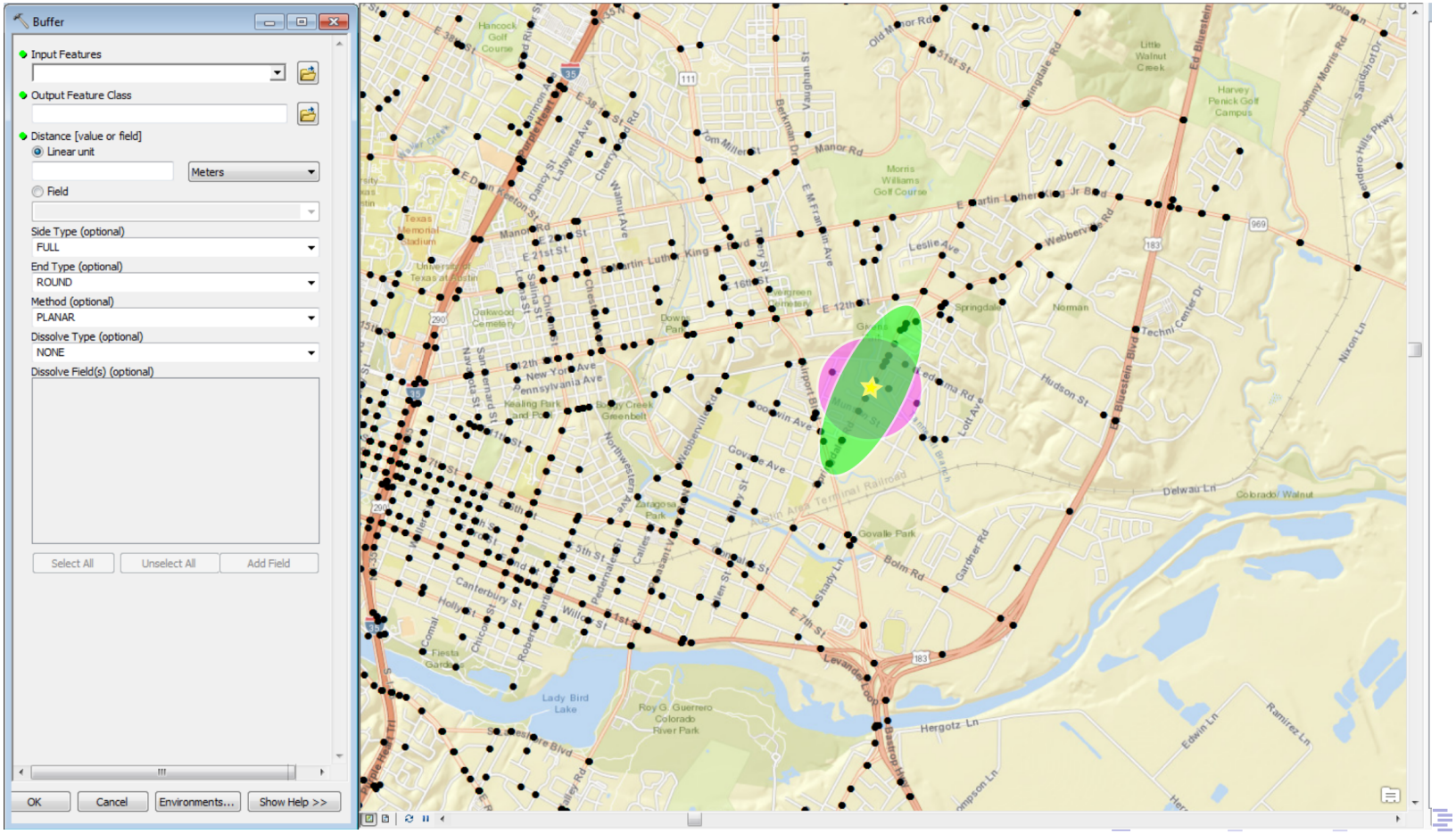
Anisotropicity Is The Norm



- Geospatial **space** is not isotropic
- Geospatial **process** is not isotropic

Buffer Tool in ArcMap

- Crime points are located along streets
- However, there is no option in ArcMap to build an **anisotropic buffer** (to estimate the location of an offender)



Spatial Statistics

- Most spatial indicators are designed based on distance only
  - E.g., Moran's I, LISA, Geary's C

$$I = \frac{N}{W} \frac{\sum_i \sum_j w_{ij}(x_i - \bar{x})(x_j - \bar{x})}{\sum_i (x_i - \bar{x})^2}$$

where w_{ij} is designed as the weight matrix comprised of pairwise distances or adjacency information

- Indicators that relate **angular effect** to spatial dependence are rare

Reasons to Assume Isotropicity

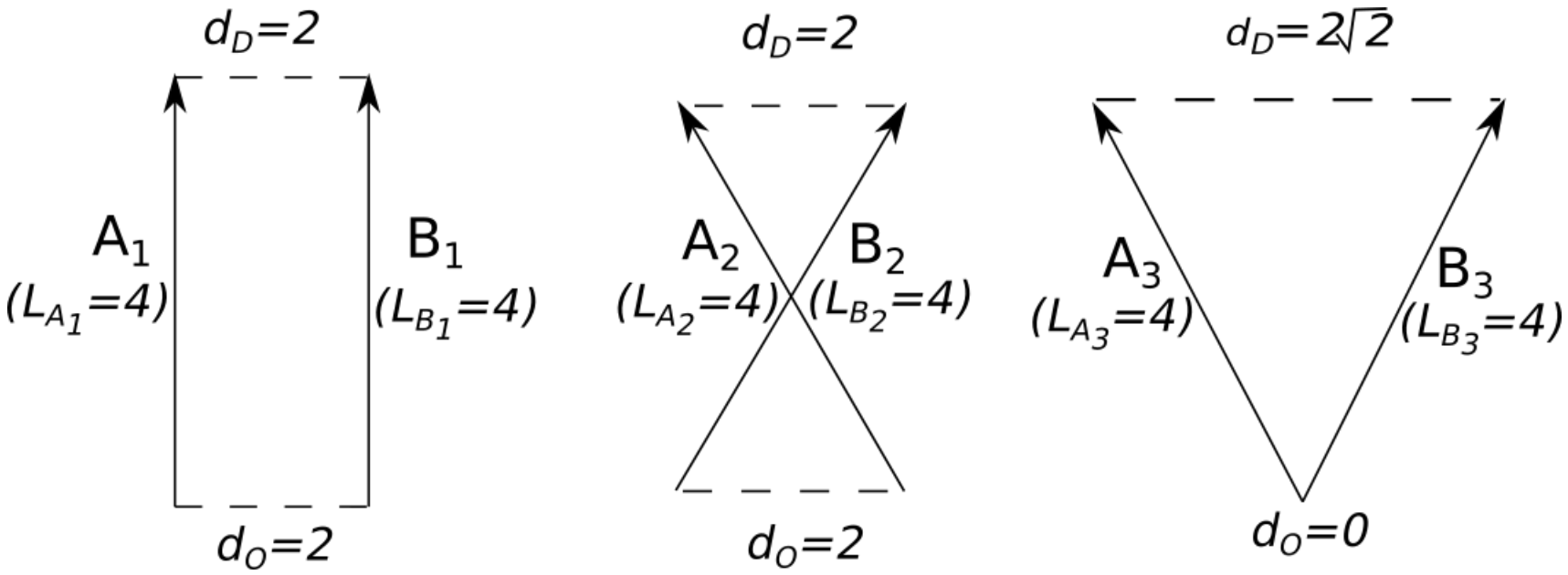
- Direction is more challenging to model
  - direction is measured on a **cyclical ratio scale**
- Direction is computationally demanding
  - direction is measured based on at least **three locations** (high ordered)
  - a **larger sample size** with enough replicates is required to conduct statistical inference
- Directional dependency varies greatly by **geographic scale**
  - Example 1: patterns of plants $\rightarrow$ large scale
  - Example 2: patterns of individual-level crime $\rightarrow$ small scale
- **Theory-induced blindness**
  - Tobler's first law of geography

Direction as an Afterthought

- Modeling spatial interactions
  - Example: similarity of spatial flows

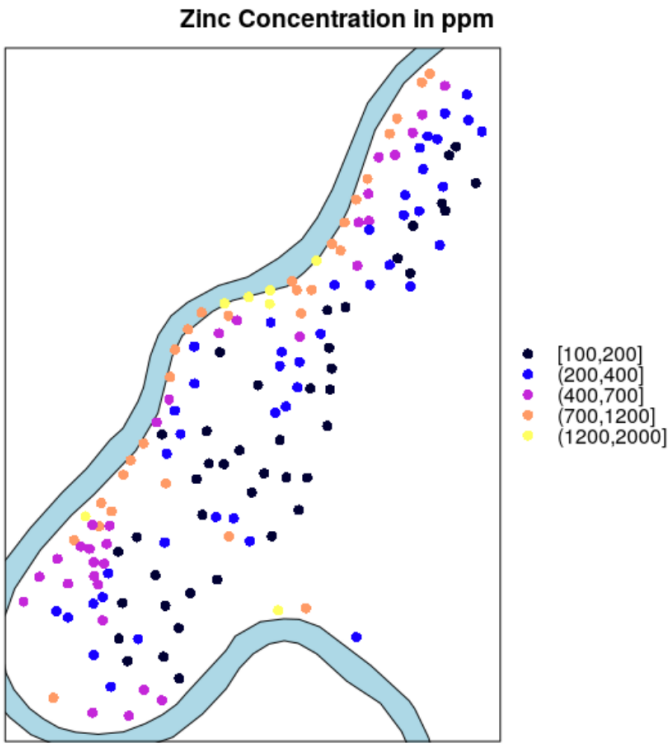
$$dis_{ij} = \sqrt{\frac{\alpha d_O^2 + \beta d_D^2}{L_i L_j}}$$

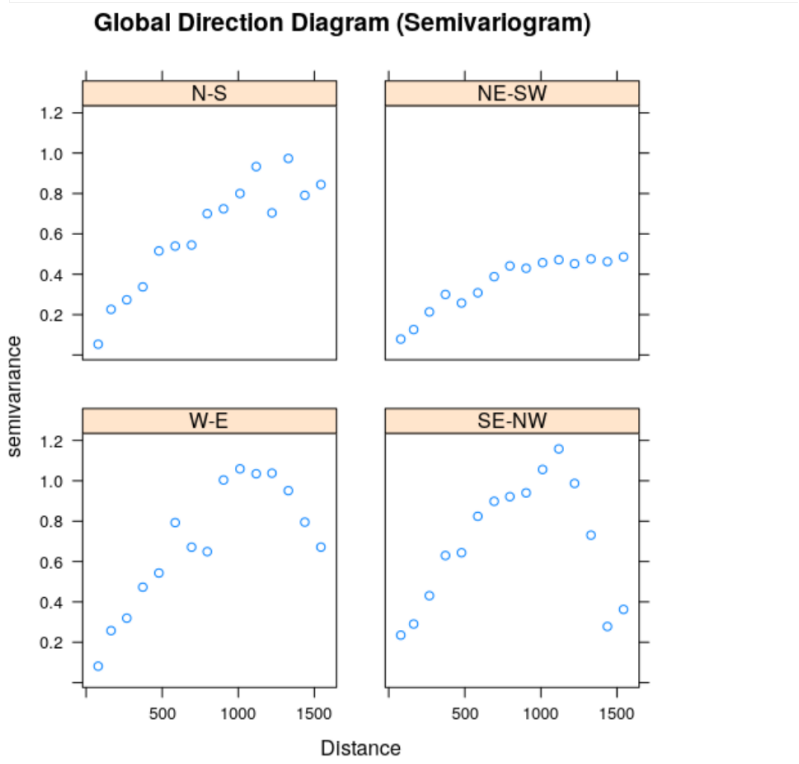
(Tao and Thill, 2016)



Direction as an Afterthought

- Modeling spatial structures
  - Example: directional semivariogram





Direction as a First-Class Citizen

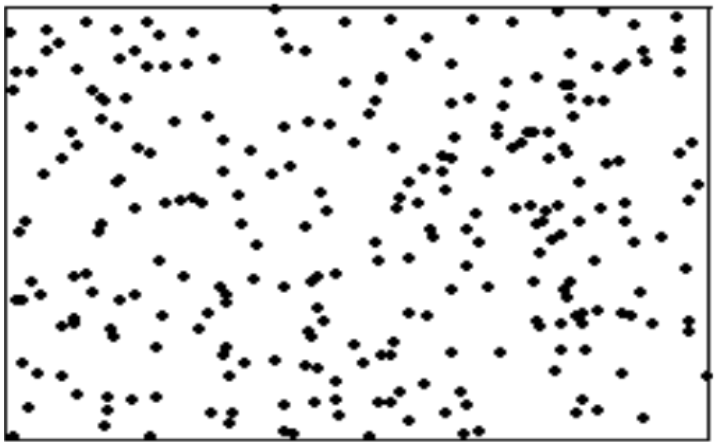
- Types of isotropicity
- Distance-free approaches

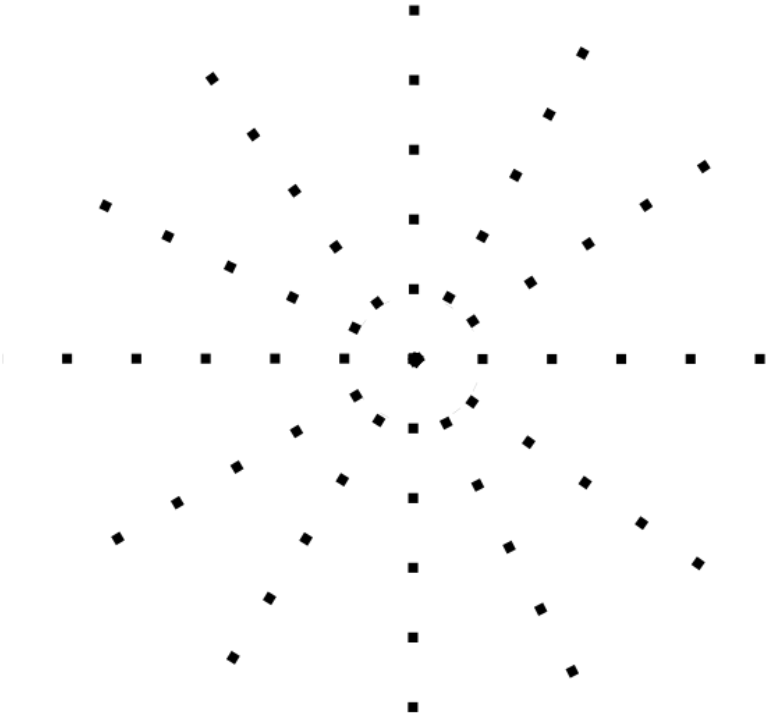
Direction as a First-Class Citizen

- **Types of isotropicity**
- Distance-free approaches

Types of Isotropicity

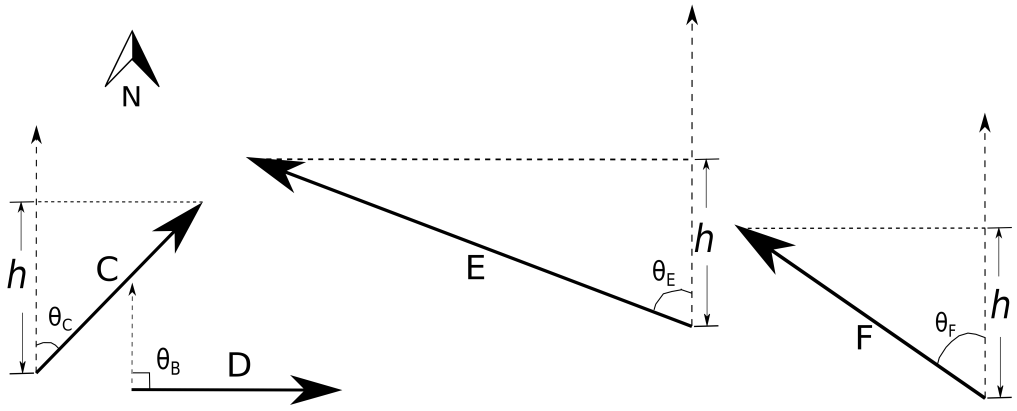
- Stationary isotropicity: isotropicity is observed **locally**
- Radial isotropicity: one **global** origin is applied



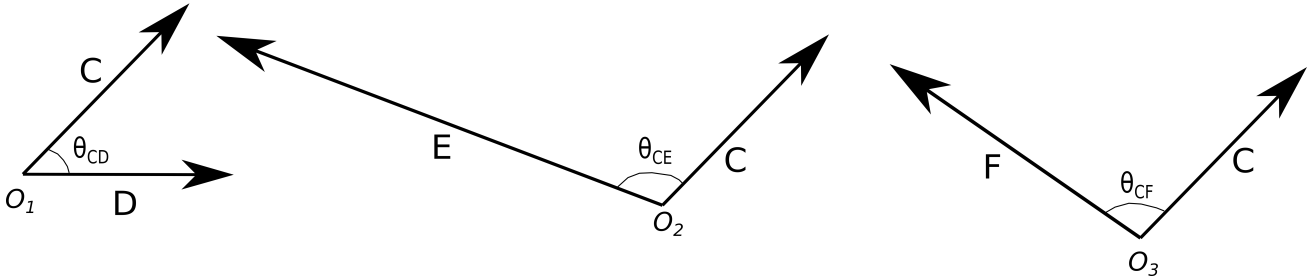


Stationary vs Radial

Radial isotropicity - Directional angle is defined globally relative to North



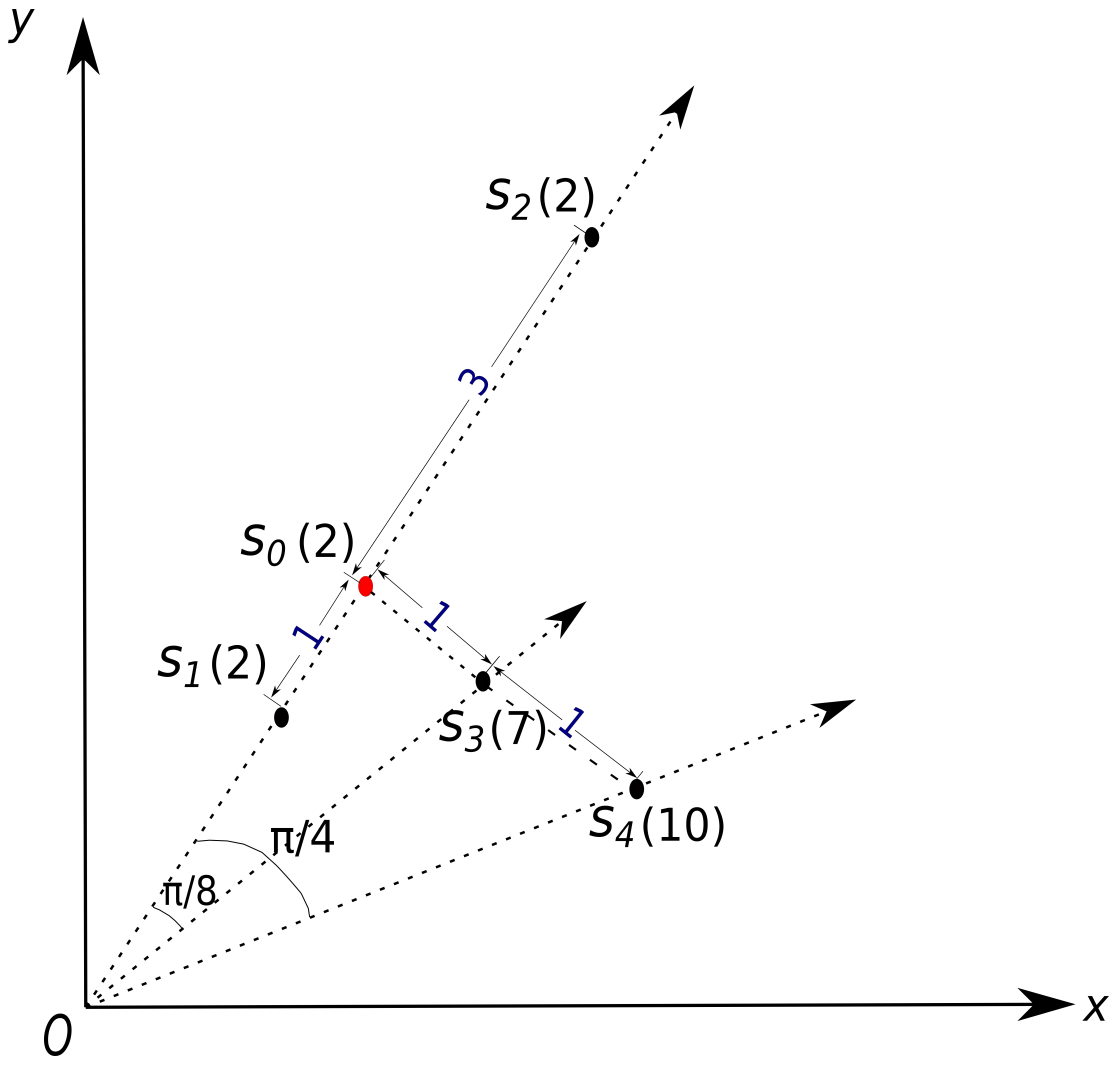
Stationary isotropicity - Directional angle is defined locally



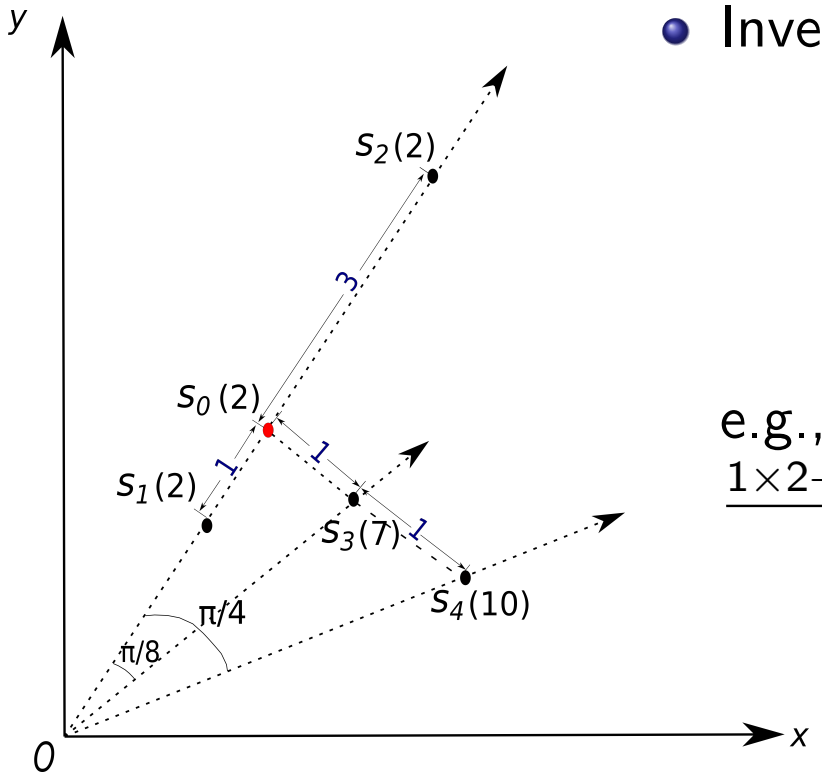
Direction as a First-Class Citizen

- Types of isotropicity
- **Distance-free approaches**

Interpolation



Interpolation - Distance-based

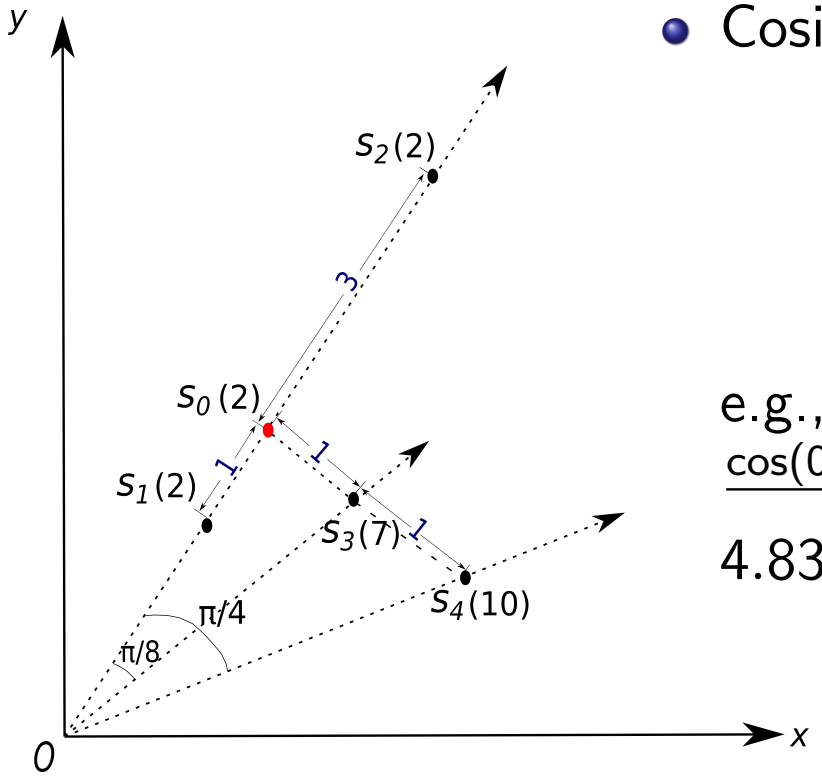


- Inverse distance weighting

$$z_i = \frac{\sum_{j=1}^{N} z_j \left(\frac{1}{d_j}\right)^p}{\sum_{j=1}^{N} \left(\frac{1}{d_j}\right)^p}$$

e.g., $z(s_0) = \frac{1\times2+(1/3)\times2+1\times7+(1/2)\times10}{1+(1/3)+1+(1/2)} = 5.17$

Interpolation - Direction-based



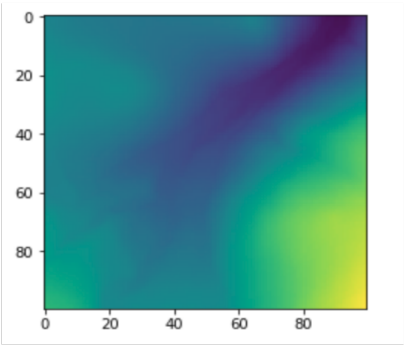
- Cosine angular weighting

$$z_i = \frac{\sum_{j=1}^{N} z_j |\cos(\theta_j)|^p}{\sum_{j=1}^{N} |\cos(\theta_j)|^p}$$

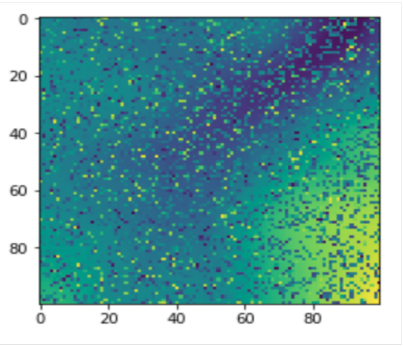
e.g., $z(s_0) = \frac{\cos(0)\times 2 + cos(0)\times 2 + cos(\pi/8)\times 7 + cos(\pi/4)\times 10}{\cos(0) + cos(0) + cos(\pi/8) + cos(\pi/4)} = 4.83$

Angular Variogram

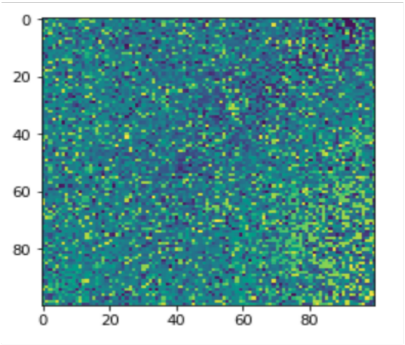
Original Pattern

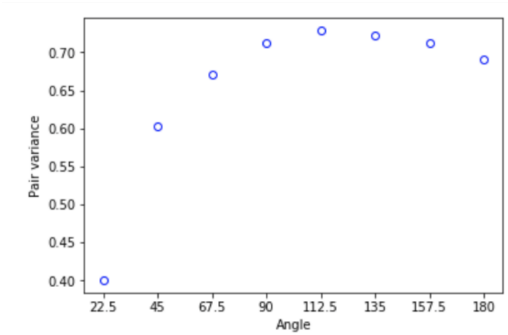


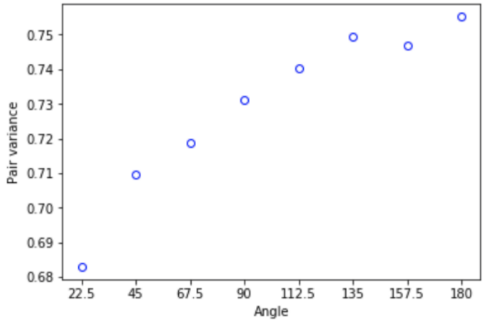
20% Randomness

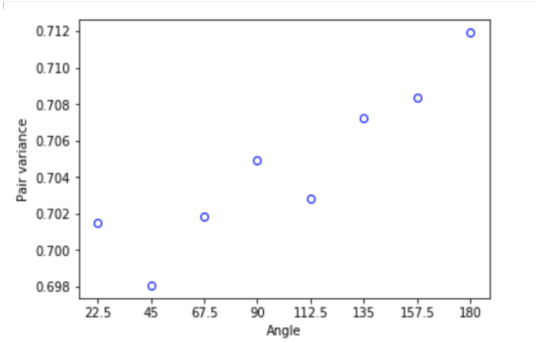


50% Randomness









Generalized First Law of Geography

Everything is related to everything else, but **near** *things and those that point* **in similar directions** *are more related than* **distant** *things and those pointing* **in different directions**.

Conclusions

- **Variance** increases with distance, but also with direction; **anisotropicity** is the norm, not the exception
- Although anisotropic techniques exist, they remain to be an **afterthought**
- **New techniques/GIS notions** can be introduced purely based on direction
- Direction can be modeled either **globally or locally**
- **Generalized TFL** highlights the role of both distance and direction in spatial analysis