Making Direction a First-Class Citizen of Tobler’s First Law of Geography

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Tobler’s First Law of Geography (TFL)

Waldo Tobler (1930 - 2018)

"Everything is related to everything else, but near things are more related than distant things" (Tobler, 1970)
The Implicit Assumption behind TFL

- Spatial relations
  - distance
  - direction
- Isotropicity
  - the **invariance** to rotations
  - $f(x') = f(Rx) = f(x)$ where $R$ is the rotation matrix
  - **directional variation** is negligible
Anisotropy is the norm

- Geospatial **space** is not isotropic
- Geospatial **process** is not isotropic
Crime points are located along streets.
However, there is no option in ArcMap to build an **anisotropic buffer** (to estimate the location of an offender).

Buffer Tool in ArcMap

![Map showing crime points and buffer tool interface]
Most spatial indicators are designed based on distance only
  - E.g., Moran’s I, LISA, Geary’s C

\[
I = \frac{N}{W} \sum_i \sum_j w_{ij} (x_i - \bar{x})(x_j - \bar{x}) \\
\sum_i (x_i - \bar{x})^2
\]

where \(w_{ij}\) is designed as the weight matrix comprised of pairwise distances or adjacency information

- Indicators that relate **angular effect** to spatial dependence are rare
Reasons to Assume Isotropcity

- Direction is more challenging to model
  - direction is measured on a *cyclical ratio scale*
- Direction is computationally demanding
  - direction is measured based on at least **three locations** (high ordered)
  - a **larger sample size** with enough replicates is required to conduct statistical inference
- Directional dependency varies greatly by **geographic scale**
  - Example 1: patterns of plants $\rightarrow$ large scale
  - Example 2: patterns of individual-level crime $\rightarrow$ small scale

**Theory-induced blindness**
- Tobler’s first law of geography
Direction as an Afterthought

- Modeling spatial interactions
  - Example: similarity of spatial flows

\[ \text{dis}_{ij} = \sqrt{\frac{\alpha d_O^2 + \beta d_D^2}{L_i L_j}} \]

(Tao and Thill, 2016)
Direction as an Afterthought

- Modeling spatial structures
  - Example: directional semivariogram

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Direction as a First-Class Citizen

- Types of isotropicity
- Distance-free approaches
Direction as a First-Class Citizen

- Types of isotropicity
- Distance-free approaches
Types of Isotropicity

- Stationary isotropicity: isotropicity is observed **locally**
- Radial isotropicity: one **global** origin is applied
Stationary vs Radial

Radial isotropicity - Directional angle is defined globally relative to North

Stationary isotropicity - Directional angle is defined locally

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Direction as a First-Class Citizen

- Types of isotropicity
- Distance-free approaches
Interpolation
Interpolation - Distance-based

- **Inverse distance weighting**

\[ z_i = \frac{\sum_{j=1}^{N} z_j (\frac{1}{d_j})^p}{\sum_{j=1}^{N} (\frac{1}{d_j})^p} \]

E.g., \( z(s_0) = \frac{1 \times 2 + (1/3) \times 2 + 1 \times 7 + (1/2) \times 10}{1 + (1/3) + 1 + (1/2)} = 5.17 \)
Interpolation - Direction-based

- **Cosine angular weighting**

\[ z_i = \frac{\sum_{j=1}^{N} z_j |\cos(\theta_j)|^p}{\sum_{j=1}^{N} |\cos(\theta_j)|^p} \]

e.g., \[ z(s_0) = \frac{\cos(0) \times 2 + \cos(0) \times 2 + \cos(\pi/8) \times 7 + \cos(\pi/4) \times 10}{\cos(0) + \cos(0) + \cos(\pi/8) + \cos(\pi/4)} = 4.83 \]
Angular Variogram

Original Pattern

20% Randomness

50% Randomness
Everything is related to everything else, but near things and those that point in similar directions are more related than distant things and those pointing in different directions.
Conclusions

- **Variance** increases with distance, but also with direction; **anisotropicity** is the norm, not the exception.
- Although anisotropic techniques exist, they remain to be an afterthought.
- **New techniques/GIS notions** can be introduced purely based on direction.
- Direction can be modeled either **globally or locally**.
- **Generalized TFL** highlights the role of both distance and direction in spatial analysis.