Grounding Geographic Categories in the Meaningful Environment

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Abstract. Ontologies are a common approach to improve semantic interoperability by explicitly specifying the vocabulary used by a particular information community. Complex expressions are defined in terms of primitive ones. This shifts the problem of semantic interoperability to the problem of how to ground primitive symbols. One approach are *semantic datums*, which determine reproducible mappings (measurement scales) from observable structures to symbols. Measurement theory offers a formal basis for such mappings. From an ontological point of view, this leaves two important questions unanswered. Which qualities provide semantic datums? How are these qualities related to the primitive entities in our ontology? Based on a scenario from hydrology, we first argue that human or technical *sensors implement semantic datums*, and secondly that primitive symbols are *definable* from the *meaningful environment*, a formalized quality space established through such sensors.

Key words: Semantic Heterogeneity; Symbol Grounding Problem; Semantic Datum; Meaningful Environment;

1 Introduction

The symbol grounding problem [13] remains largely unsolved for ontologies: ultimately, the semantics of the primitive terms in an ontology has to be specified outside a symbol system. Tying domain concepts like river and lake to data about their instances (as proposed, for example, in [5]) constrains these in potentially useful ways, but defers the grounding problem to the symbol system of the instance data. While these data may have shared semantics in a local geographic context, they do not at higher levels, such as in an INSPIRE scenario of integrating data and ontologies across Europe (`http://inspire.jrc.ec.europa.eu/`). It does not seem practical for, say, Romanian hydrologists, to ground their lake and river concepts in British geography, or vice versa. Furthermore, grounding domain concepts in a one-by-one manner is an open-ended task. One would prefer a method for grounding ontological primitives in *observation procedures* in order to support more general ontology mappings.

In this paper, we propose such a method and demonstrate its applicability by the category *water depth*. We provide an ontological account of Gibson's meaningful environment [12] and use Quine's notion of observation sentences [20] as a basis for grounding ontological concepts in reproducible observation procedures. At first glance it seems improbable that highly elaborated scientific concepts, like those of INSPIRE, could be reconstructed from meaningful primitives. Although such a wider applicability of the method remains to be shown, we suggest however - like Quine - that even the elaborated language of natural science must eventually be grounded in observational primitives. After discussing basic issues from measurement theory, philosophy and cognition (section 2), we review the core ideas of Gibson's meaningful environment and formalize them (section 3). Using the example of water depth, we apply the theory in section 4, before drawing some conclusions on what has been achieved and what remains to be done (section 5).

2 Measurement and the Problem of Human Sensors

In this section we introduce the notion *semantic datum* and claim that successful grounding ultimately rests on the existence of human sensors for body primitives.

2.1 Semantic Datums for Languages about Qualities

Measurement scales are maps from some observable structure to a set of symbols [26]. Measurement theory merely provides us with formal constraints for such mappings, namely scale types. It does not disambiguate scales themselves. For example, we can distinguish ratio scales from interval scales, because ratio scales can be transformed into each other by a similarity transformation, while for interval scales we need a linear transformation [26]. But individual scales are never uniquely determined by their formal structure. This is called the *uniqueness problem* of measurement [26]. Therefore the symbol grounding problem [13] remains unsolved: In order to disambiguate scales, we need to know about the conventions of *measurement standards*, like unit lengths or unit masses.

One approach to this problem are semantic datums [14][18]. A semantic datum interprets free parameters. It provides a particular interpretation for the *primitive symbols* of a formal system. An interpretation is a function from all symbols (terms, attributes, and relations) in a formal symbol system to some particular other structure which preserves the truth of its sentences. Typically, formal systems allow for more than one interpretation satisfying their sentences, and therefore they have an ambiguous meaning. As non-primitive symbols in a formal system are definable from the primitive ones, a semantic datum can fix one particular interpretation. The structure in which the symbols are interpreted can be either *other formal systems* (*reference frames* in the sense of Kuhn et al. [15]) or *observable real world structures (qualities)*. We say that a formal system is *grounded*, if there exist semantic datums for an interpretation into qualities. Examples for such semantic datums are *measurement standards*. A formal system may also be *indirectly grounded* by chaining several semantic datums. Simple examples of grounded formal systems are *calibrated measurement devices*, like a thermometer. A semantic datum is given by the observable freezing and boiling events of water at a standard air pressure. More complex examples are *datums for geodetic positions*: A directly observable semantic datum for the positions on a Bessel ellipsoid consists of a named spot on the earth's surface like "Rauenberg" near Berlin (*Potsdam Datum*) and a standard position and orientation for the ellipsoid.

How can we expand these ideas to arbitrary languages about qualities? Better: For which language primitives do such semantic datums exist?

2.2 Sensors as Implemented Scales

In this paper, we consider a sensor to be a device to *reproducibly* transform observable structures into symbols. A sensor implements a measurement scale including a semantic datum, and therefore establishes a source for grounding. The main requirement for such sensors is reproducibility, i.e. to make sure that multiple applications of the sensor assign symbols in a uniform way.

We already said that *calibrated measurement devices* are grounded formal systems, and therefore we call them *technical sensors.* Following Boumans [8], any reliable calibration of a technical sensor is based ultimately on *human sensation,* because it needs *reproducible gauging* by human observers [8].

So any grounding solution based on measurement ultimately rests on the existence of *human sensors.* But what is a human sensor supposed to be? For human sensors, reproducibility means something similar to *inter-subjective word meaning* in linguistic semantics. More specifically, we mean with a human sensor that an information community shares symbols describing a *certain commonly observable situation.*

2.3 Are there Human Sensors for Body Primitives?

Embodiment and Virtuality of Language Concepts A commonly observable situation is exactly what Quine [20] described with *occasion sentences* and more specifically with *observation sentences.* Quine's argument is that natural language sentences vary in their semantic indeterminacies. There are certain occasion sentences, utterable only on the occasion, with relatively low indeterminacy and high *observationality.* These sentences are called *observation sentences.* Symbols of a language in general inherit their meaning from such sentences, but the further away they are from such observation sentences inside of a language, the more abstract and indeterminate they get. Thus a symbol like "Lake Constance" (a name for an individual) is less virtual than "Lake" (a general term), which is again less virtual than a social construction like "Wetland". Whether a symbol is less virtual than another is primarily dependent on its reference to *bodies and their parts.* This is what Quine called *divided reference* ([20], chapter 3): Humans individuate bodies like "Mama" by reference to (pointing at) their observable parts and using a criterion of individuation. And they quantify over general terms (categories) like "Mother" by reference to a yet undetermined number of similar but virtual bodies.

The empirical arguments from *cognitive linguists*, e.g. [16], that embodiment is the semantic anchor for more virtual language concepts via metaphors, seem to underpin these early ideas about body based primitives. Langacker [16] suggests that *imagined bodies and fictive entities* are a semantic basis for formal logic and quantifier scopes in the sense of Quine.

Following these lines of thought, we take the view that language semantics, especially the semantics of formal ontologies, is anchored in the individuation of bodies as unified wholes of parts of the environment. Individuation rests on perceivable qualities, e.g. their shape. But how can we imagine humans to perceive such properties of bodies and their parts?

Scanning A certain kind of virtuality in perception is especially interesting for us. Drawing on ideas of Talmy, Langacker [16] also suggests that *mental scanning*, a fictive motion of a virtual body in the perceived or imagined environment, is central to language semantics. The sentence "The balloon rose quickly" thus denotes an actual body movement, whereas "This path rises quickly near the top of the mountain" can only be understood by imagining a virtual body movement in an actual environment. This view is also supported by recent work on grounded cognition [1] which claims that human cognition is grounded through situated simulation. The motion oriented notion of scanning proposed here is a special case of such situated simulation. We assume that *some perception* is *scanning*: *a series of virtual steps in an environment*, with each step leading from one *locus of attention* to the next.

Ostension and Agreement on Names In order to assume a human sensor for an observable language symbol, it is necessary that different actors (as well as the same actor on different occasions) will reliably agree on the truth of its observation sentences in every observable situation. This effectively means that there must be consensus about names for bodies and body parts. Quine [20] suggested that *observable names*, like "Mama", can be agreed upon in a language community by *pointing* at a body. According to Quine, the agreement on names for bodies can be based on an observable action such as ostension, given the situation is simultaneously observed and the viewpoints of a language teacher and a learner are enough alike. In the same manner, the correct word usage is inculcated in the individual child of a language community by social training on the occasion, that is by the child's disposition to respond observably to socially observable situations, and the adults disposition to reward or punish its utterances[1] ([20], chapters 1 and 3). In this way, agreement on names for bodies actually spreads far beyond the concretely observable situation, involving a whole language community. *Ostension* is nothing else than a communicative act including a virtual movement, because an observer has to scan a pointing body part, e.g. a finger, and extend it fictively into space. So this fits our assumptions.

[1] According to Quine, observation sentences are the *entrance gate* to language, because they can be easily learned directly by ostension without reference to memory

Further on, we will just assume that body parts and body primitives can always be given unambiguous names by using ostension.

3 Gibson's Meaningful Environment

In this section we discuss a formal grounding method based on Gibson's meaningful environment. We begin with two examples illustrating perceptual primitives for bodies and then proceed with a discussion of Gibson's ideas.

3.1 The Blind Person in a Closed Room

How does a blind man perceive the geometric qualities of a closed room? Standing inside the room, he can rely on his tactile and hearing sensors to detect its surface qualities. Because he knows his body takes some of the space of the room, the room must be higher than his body. If he can turn around where he stands, he knows that a roughly cylindrical space is free and part of the room. By taking a step forward, he concludes that an elongated "corridor" is free and part of the room. Because he can repeat steps of the same length into the same direction, he can step diametrically through the room and even measure one of its diameters. His last step may be shortened, because his foot bumps into the wall. He thus detected the inner surface of the room. If he continues summing up paths through the room, he can individuate the whole room by its horizontal extent.

3.2 The Child and the Depth of a Well

Imagine a child in front of a water well trying to assess its depth. It cannot see the ground in the well as no light reaches it. Nevertheless, the child can perform a simple experiment. It drops a brick from the top of the well and waits until it hits the water surface. The child cannot see this happening but can hear the sound. It can repeat the experiment and count the seconds from dropping to hearing the sound, and hence it can measure the depth of the well. The child assesses the depth by simulating a motion that it cannot do by its own using a brick.

3.3 A Short Synopsis and Extension of Gibson's Ideas

Gibson [12] sketches an informal ontology of elements of the environment that are accessible to basic human perception, called *the meaningful environment.* In the following, we try to condense his ideas about what in this environment is actually directly perceivable and complement them with the already discussed ideas about fictive motion.

The environment is mereologically structured at all levels from atoms to galaxies. Gibson claims that at the ecological scale, so called *nested units* are basic for perception: Canyons are nested within mountains, trees are nested within canyons, leaves are nested within trees. The structure of ecological units depends on the environment as well as the perceiving actor in it. But the perception of individual units, of their composition and of certain aspects of their form and texture are common to humans in a certain ecological context. Thus we can assume a *human sensor* for them. Although there are no a priori atomic units, *perceptual limits* do exist for geometric properties (we will call this *perceivable granularity*). Biological cells are beyond these limits, and therefore not directly perceivable, whereas leaves are.

The composition of ecological units determines the *surface qualities of meaningful things*, i.e. their shape, called *layout*, and their *surface texture* (including colors). Furthermore, these surface qualities *individuate the meaningful things* in the environment by *affordances*. Due to ecological reasons, things with surfaces are able to afford actions: for example to support movements, to enclose something as hollow objects, to afford throwing as detached objects. Surface perception is therefore considered by Gibson to be a *reliable mechanism for object individuation*: Surfaces are the boundaries of all meaningful things humans can distinguish by perception. Beyond each surface lies another meaningful thing (*exclusiveness*), and the meaningful environment is *exhaustively* covered by such meaningful things (there is no part of it that is not covered by them). Furthermore, all major categories of meaningful things can be individuated by some affordance characteristic based on surface qualities.

The most important top level categories of such things are *substances*, *media and surfaces*. A *part of a medium* is a unit of the environment that affords *locomotion through it*, is *filled with illumination* (affords seeing) and *odor* (affords smelling) and bears the *perceivable vertical axis of gravity* (affords vertical orientation). In this paper, we will restrict our understanding of a medium to locomotion affordances. It is clear that the classification of a medium is stable only in a certain *locomotion context*: water is a medium for fish or divers, but not for pedestrians. So there will be different media for different locomotion contexts, but for most cases, including this paper, it will be enough to consider two of them: *water* and *air*. *Substances* simply denote the rigid things in a meaningful environment that do not afford locomotion through them. *Surfaces* are a thin layer of medium or substance parts located exactly where any motion must stop.

We complement Gibson's ideas by drawing on the concept of *virtual places and fictive motion* outlined in 2.3. In doing so, we reaffirm the idea of affordances as central to the perception of the environment, because we assume that substances, media and surfaces can be perfectly conceived through the imagination of virtual bodies moving through them. So if people say that the branches of a biological tree are thicker than their forearm, we consider them conceiving parts of the tree and the forearm as *places for a virtual body*. We call such a place *locus of attention*. Furthermore, we also consider humans being able to refer to parts of places that are *beyond perception* and therefore even more virtual: Humans can refer for example to the cells of a leaf without perceiving them. We closely stick to the idea that this perceivable environment is the *source* for human conceptualization, and all other categories are *refinements* or *abstractions* from them. In particular, our notion of *place* is e.g. *much less abstract* than Casati and

Varzi's notion [9]: Entities and their places are not distinguishable, and therefore two things occupying the same place are the same. This also distinguishes our approach from that of an ecological niche [22].

3.4 Nested Places

The central methodological question is: for which structures in a Gibson environment can we assume sensors, and what are their formal properties? In this section, we will introduce and discuss the domain G (denoting the domain of virtual places) and the part-of relation $P[partof]$ on G. In Sect. 3.5, 3.6 and 3.7, we discuss geometrical properties of virtual steps, $Step$, $=_L$ $[equallength]$ and $OnL[equaldirection]$ in G, and in sections 3.8 and 3.9 we introduce medium connectedness $AirC$ and $WaterC$ and verticality $VertAln$ on G, respectively. In the formal part of these sections, we introduce a first order theory and assume for convenience that all quantifiers range over G unless indicated otherwise, and that all free variables are implicitly all-quantified.

Following our discussion above, we take the view that our *domain* G consists of *nested places for the actual and virtual things that can be perceived* in the environment. Mathematical artifacts like infinitely thin planes, lines and points are not in this domain, because they cannot contain extended things and are not perceivable. So we must construct the whole environment from something equivalent to regular regions in Euclidean space. Places have well-behaved mereological and geometric structures, which will be discussed in the subsequent sections. Our ideas about this structure were influenced by [2], [6].

As discussed in Sect. 3.3, our domain of places has a part-whole structure humans can refer to by pointing, and this structure is assumed to be *atomless* (even though we assume a granularity for perceiving their geometrical or topological properties):

Axiom1 We assume the axioms of a *closed extensional mereology* (CEM) [9] for a primitive part-of relation $P : G \times G$ on places G (meaning the first place is a part of the second), so that the mereological sum of every collection of places is another existing place and two places having the same parts are mereologically equal. The usual mereological symbols PP (proper-part-of), O (overlap), PO (proper overlap) and $+$ (sum) are definable.

3.5 Steps and their Length and Direction

We suppose that humans experience the geometrical and topological structures of the meaningful environment by a primitive binary relation $Step(a, b)$, meaning the *virtual or actual movement of a locus of attention* from place a to b. Humans perceive *length* and *direction* of steps, because (in a literal sense) they are able to repeat steps of equal length and of equal direction. And thereby, we assume, they are able to observe and measure lengths of arbitrary things in this environment. The ratio scale properties of these lengths, as described e.g. by extensional systems in [26], must then be formally derivable from the structural properties of steps. What formal properties can we assume for such perceivable steps?

The visual perception of *geometrical properties*, like equality of distances and straightness of lines, is a source of puzzles in the psychological literature, because human judgment tests revealed e.g. that perceived straight lines are not equivalent to the usual Euclidean straight lines [23]. Like Roberts et al. [21] suggested, it is nevertheless plausible to assume that the usual *Euclidean properties*, like congruence of shapes under rotation, are reconstructed by humans through learning. We adopt this view because Euclidean properties seem indispensable for human orientation and the recognition of body concepts.

Perceived geometry should have a *finite* and *discrete* structure, because of the *resolution properties* of sensors [10], and human perception in particular [17]. There are *finite approaches to geometry* available that seem to fit well to our problem (see Suppes [24]), but would also require finite and approximate accounts of a length scale (an example for such a scale can be found in [25]).

As a first step, we confined ourselves in this paper to an *infinite and dense version of a theory of steps with granularity*, based on the pointless axiomatization of *Euclidean geometry* given originally by Tarski [27]. We write $xy =_L uz$ for two steps from locus x to y and from u to z having *equal length,* and $OnL(x, z, y)$, if locus z is *on a line* between x and y or equal to any of them (compare Fig. 1). Note that OnL implies collinearity and betweenness. For the rest of the paper, we assume that the primitives OnL and $=_L$ are *only defined for loci of attention.*

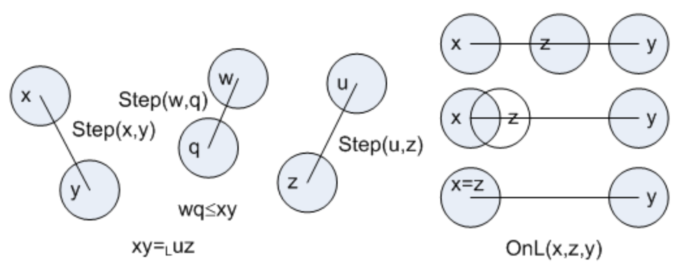

Fig. 1. Equal length and equidirection for steps.

Our equidistance $=_L$ and equidirection OnL primitives for steps satisfy a 3-D version of Tarki's *equidistance* and *betweenness* axioms for *Elementary Geometry* [27], similar to the approach in [2]. The quantifiers on points in Tarski's version or on spheres in Bennett's version can be replaced by quantifiers over the domain and range of steps: Our quantifier $\forall^{Locus} x.F(x)$ for example, meaning $\forall x.Step(x, x) \rightarrow F(x)$, restricts the domain of places to the *loci of attention.* Unlike Tarski and Bennett [2], we do not assume sphericity but the step relation as a primitive. Furthermore, *identity of points* in Tarski's or *concentricity of spheres* in Bennett's version just means to *"step on the spot"* in our theory. We therefore assume a version of Tarski's axioms with identity of points replaced by mereological equality of loci of attentions:

Axiom2 We assume the following axioms for *equidistance* $=_L$: Symmetry, identity, transitivity (compare the three axioms for *equidistance* in [2]), and for *equidirection* OnL: Identity, transitivity and connectivity (compare the three axioms for *betweenness* in [2]). We also assume the axioms of Pasch, Euclid, the Five-Segment axiom, the axiom of Segment Construction, the Weak Continuity axiom, and a 3-D version of the Upper and Lower Dimension axiom, as described in [2].

Tarski's axioms ensure that there are loci of attention centered at all the "points" of a Euclidean space. It follows that steps and their lengths and direction have the expected Euclidean properties, and in particular that *each pair of loci of attention* forms a step with these properties.

3.6 Loci of Attention

A locus of attention is a smallest perceivable place. It can be thought of as a granular sphere with congruent shape giving rise to a minimal resolution for geometric properties in general. This is because we assume that humans perceive the geometrical qualities of arbitrary places by covering them with loci of attention. In fact, loci of attention are our simplified version of *just noticeable differences* in psycho-physics [17], but being independent of the stimulus.

We can define a notion of *shorter than* $\leq$, holding iff a step from y to x is shorter than a step from q to z:

Definition 1. ***(shorter than)*** $\forall^{Locus} y, x, q, z.(yx \leq qz) \leftrightarrow \exists x'.OnL(y, x, x') \wedge yx' =_L qz$

We now can define a topological notion of *touching* or *weakly connected*, which applies for a smallest step with non-overlapping loci:

Definition 2. ***(touching)*** $\forall^{Locus} x, y.Touching(x, y) \leftrightarrow \neg O(x, y) \wedge (\forall z.Step(x, z) \wedge xz \leq xy \rightarrow O(x, z))$

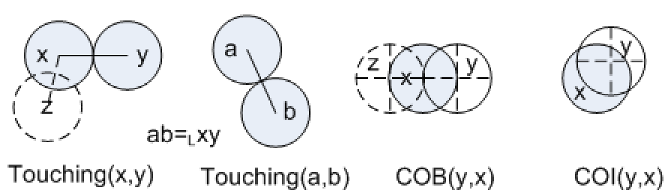

Fig. 2. Touching steps, steps centered on boundary (COB) and interior (COI).

All loci of attention are required to be *congruent* to each other. This can be expressed by requiring congruent lengths for all pairs of touching loci:

Axiom3 ***(locus of attention)*** $\forall^{Locus}x, y, z, u.Touching(x, y) \wedge Touching(z, u) \rightarrow xy =_L zu$

It follows that loci are spheres of a fixed size. Similar to [3], *significant places* are the ones that are big enough to contain a granular locus of attention:

Definition 3. ***(significant place)*** $Significant(r) \leftrightarrow \exists^{Locus}x.P(x, r)$

We hence assume that insignificant places fall beyond the perceivable resolution for geometric properties. A *discrete* theory of steps would allow for an even stronger notion of resolution based on a minimal step length (see [24]).

3.7 The Environment is Wholly Covered by Steps

A step is the central perceivable relation giving rise to affordances, because if we *virtually step through an environment,* it affords the *continuous transfer* of a virtual body from one place to another. This is only possible if those places are *strongly connected*. A *strongly self-connected* place always contains a *sphere* which, when we split the place at any point, overlaps both halves of the split (compare definitions in [4], [7]). So there is a 2-D surface in the middle corresponding to any cut (like "cutting in wood"), and not a line or a point. Strong connectedness can be expressed based on our primitives by introducing *paths*:

We call the minimal elongated place one can step in a *path*. We assume that if there is *a step from locus x to y on a path p*, then x and y are part of p, and there is always exactly one other locus z on p with equal distance from x and y :

Definition 4. ***(path)*** $EndsOfPath(x, y, p) \leftrightarrow Step(x, y) \wedge P(x + y, p) \wedge$
$(\exists!z.Step(x, z) \wedge P(z, p) \wedge xz =_L zy) \wedge (\neg\exists p'.EndsOfPath(x, y, p') \wedge PP(p', p))$

The idea is that *paths* are the smallest elongated places of minimal thickness enclosing a step and all the closer steps in between them. Because there is exactly one locus z in the middle of x and y on the path p, we assure that the path has minimal thickness and is elongated (see Fig. 3). Because there is no smaller path with that property, we make sure that x is the beginning and y is the end of the path. The definition implies strong connectedness of a path, because all pairs of loci have a continuous collection of loci in between them, and loci are spherical.

A general definition of strong connectedness for significant places is then straightforward:

Definition 5. ***(strong connectedness)*** $SC(x) \leftrightarrow Significant(x) \wedge$
$(\forall^{Locus}u, z.P(u + z, x) \rightarrow \exists p.EndsOfPath(u, z, p) \wedge P(p, x))$

We take the view that our domain of places, the meaningful environment, is *wholly covered by steps and paths*, because we assume that this is the common way how people experience their environment. It turns out that our environment of places is quite similar to the ideas outlined in [4], [2], [6], especially Bennett's region based geometry RBG. In order to establish the link between steps on one side and arbitrary significant places on the other, we will introduce 4 axioms

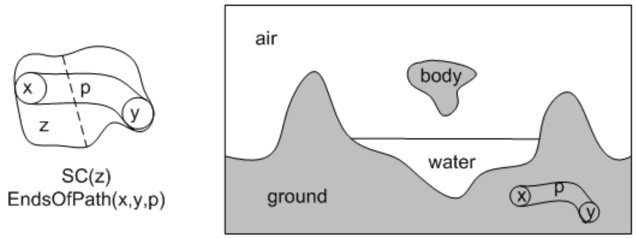

Fig. 3. Strongly connected places are connected by paths. The meaningful environment is wholly covered by strongly connected places.

along the lines of thought in [4], which ensure that each significant place coincides with a set of centers of loci of attention.

To this end we need a topological notion called *centered on the boundary* *COB(y,x)* (compare Fig. 2), meaning y is just on the boundary of x,

Definition 6. ***(centered on boundary)*** $\forall^{Locus} y, x.COB(y,x) \leftrightarrow (\exists z.Step(x,z) \wedge OnL(z,x,y) \wedge (zx =_L xy) \wedge Touching(z,y))$

from which a further notion, *centered on the interior* (Fig. 2), is definable.

Definition 7. ***(centered on interior)*** $\forall^{Locus} y, x.COI(y,x) \leftrightarrow (\exists z.Step(x,z) \wedge COB(z,x) \wedge (\neg zx =_L xy) \wedge (xy \leq xz))$

For *arbitrary places of a significant size* x we can now define a predicate meaning that a step is centered on its interior.

Definition 8. ***(centered on interior)*** $\forall^{Locus} y.COI(y,x) \leftrightarrow (\exists z.Step(y,z) \wedge COI(y,z) \wedge P(z,x))$

We first have to make sure that there is a sum of loci of attention corresponding to every (significant) open 3-ball in our Euclidean environment. We therefore assume that for each step xy of at least half a locus length, there is a perceivable ball z centered on x and topologically bounded by y. This is actually a granular variant of Bennett's Axiom 6 in [4]:

Axiom4 ***(steps give rise to significant 3-balls)*** $\forall^{Locus} x, y.\neg COI(x,y) \rightarrow (\exists z.(\forall w.COI(w,z) \leftrightarrow (xw \leq xy \wedge \neg xw =_L xy)))$

We also assume that the domain of steps is *extensible*, so there are always larger balls constructible (compare Axiom 7 in [4]):

Axiom5 ***(steps are extensible)*** $\forall^{Locus} x, y.\exists z.Step(x,z) \wedge (x \neq z) \wedge (\forall z'.xz \leq xz' \leftrightarrow (xy \leq xz' \wedge \neg xz' =_L xy))$

Secondly, we have to make sure that center points on *arbitrary significant places* behave in correspondence with their mereological structures. So parts of places always imply interior steps (compare Axiom 8 in [4]):

Axiom6 ***(parts imply interior steps)*** $P(x, y) \leftrightarrow (\forall^{Locus} u.COI(u, x) \rightarrow COI(u, y))$

And third, we must assure that all places are actually covered by steps (compare Axiom 9 in [4]):

Axiom7 ***(places overlap with loci of attention)*** $\forall r \exists^{Locus} x.O(x, r)$

As was shown by Bennett [4], our Axioms 1-7 provide an axiom system for 3-dimensional regular open sets of Euclidean space: It can be proven that the sets of interior loci ("points") of arbitrary sums of loci are *regular open*. Because of granularity of perception, places are not in general constructible from steps in our theory. It can nevertheless be proven that they are *coverable* by loci of attention, and these covering sums of loci must have the expected geometrical properties:

Proposition 1. *Every place is part of a sum of loci of attention.*

To see this, be aware that from Axiom 7 it follows that every place has a part that is part of a locus of attention. With Axiom 1 we assume that every place is a sum of such parts. Thus every place is coverable by steps.

3.8 Media and Substances are Wholes under Simple Affordance

Closely following Gibson, we assume that humans can directly perceive whether the environment *affords a certain type of movement*, and are thereby able to *individuate bodies, media and their surfaces*. Like in the previous section, we can think of such movements as fictive motions, and therefore the affordance primitives in this section are just refinements of our already introduced *step* primitive. In general, we assume that humans can perceive a multitude of *such simple affordances*. For our purpose, we will describe two of them, $AirC$ and $WaterC$, for movement in *air* and *water*, respectively. Because they have the power of individuation, we assume that those relations are *mutually exclusive*. Then we can *define media* as *unified whole*s under the respective affordance primitive.

Media. Connected by the same medium implies a step on a medium-connected *path* and is *symmetric* and *transitive*:

Axiom8 ***(connected by the same medium)*** $MediumC(x, y) \rightarrow \exists p.EndsOfPath(x, y, p) \wedge (\forall^{Locus} z.P(z, p) \rightarrow MediumC(z, x) \wedge MediumC(z, y)) \wedge (\forall u.MediumC(y, u) \rightarrow MediumC(x, u))$

The actually observable primitives are its two mutually exclusive sub-relations $AirC$ and $WaterC$,

Definition 9. ***(a medium is either air or water)*** $MediumC(x, y) \leftrightarrow AirC(x, y) \vee WaterC(x, y)$

Axiom9 ***(mutual exclusiveness)*** $MediumC(x,y) \wedge MediumC(u,z) \wedge (z+u)O(x+y) \rightarrow ((WaterC(z,u) \wedge WaterC(x,y))Xor(AirC(z,u) \wedge AirC(x,y)))$

which give rise to media *water* and *air* by using them as *unity criterion*: *A (water/air) medium* is any maximal medium-connected whole:

Definition 10. ***(media)*** $Air(x) \leftrightarrow Whole(x, AirC) \wedge$
$Water(x) \leftrightarrow Whole(x, WaterC) \wedge Medium(x) \leftrightarrow Whole(x, MediumC)$

For a definition of *whole* as a *maximal sum* of parts *connected* by a partial equivalence relation, we refer to [11]. Informally, a medium is just any place which has all places of an *equivalence class* of *AirC* or *WaterC* as parts. Due to Axiom 8 and by definition, media must be composed of paths and therefore must have a significant size.

Substances and Bodies. According to Gibson, substances are not directly perceivable, only via the perception of a certain medium and its surface: just like walls are only perceivable as obstacles for the locomotion of light and other bodies through media. In this view, substances can be defined from media and define all other forms of places:

Definition 11. ***(substance)*** $Substance(x) \leftrightarrow \neg\exists z.(Medium(z) \wedge O(xz))$

Nevertheless, humans can obviously recognize the shape of certain significant strongly connected substances, called *bodies*. Therefore we must conceive bodies, similar to media, as individual wholes made up of *substance paths*. This is possible because the whole environment is covered with virtual paths by proposition 1. Bodies are maximal strongly connected substance wholes:

Definition 12. ***(connected by a body)*** $BodyC(x,y) \leftrightarrow Substance(x+y) \wedge SC(x+y)$,

meaning two places are connected by the same body if they are substances and their sum is strongly connected, and

Definition 13. ***(body)*** $Body(x) \leftrightarrow Whole(x, BodyC)$,

meaning a body is any maximal strongly-connected whole of substances.

Surfaces. Surfaces must also be definable using steps, and therefore have a *minimal thickness*. Thereby we avoid the philosophical question whether an infinitely thin topological boundary belongs to a region or its complement (see Casati and Varzi [9]). We take Borgo's view advocated in [7] and assume that every individuated body or medium has its own surface, which is simply a thin layer of paths making up the surface part of the body or medium (compare Fig. 4), such that every step's locus is touching a locus from the outside of the body or medium:

Definition 14. ***(connected by a surface)*** $SurfaceC(x,y,r) \leftrightarrow$
$(Medium(r) \vee Body(r)) \wedge (\exists p.EndsOfPath(x,y,p) \wedge P(p,r) \wedge (\forall^{Locus} z.P(z,p) \rightarrow (\exists^{Locus} u.\neg O(r,u) \wedge Touching(z,u))))$

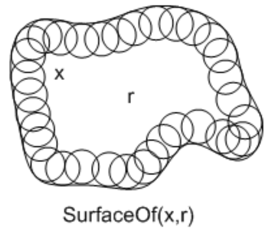

Fig. 4. Surfaces of bodies consist of a thin layer of virtual paths touching the outside of the body.

A surface is then a maximal surface connected part of a body or a medium:

Definition 15. ***(surface)*** $SurfaceOf(x, r) \leftrightarrow$
$Whole(x, (\lambda u, v.SurfaceC(u, v, r)))$

From these definitions and Axioms 8 and 9 it is provable that substances and media in fact make up the whole meaningful environment:

Proposition 2. *Substances and Water and Air "partition" the meaningful environment: Any place that is part of one category cannot be part of another one (mutual exclusion), and any place is part of a sum of places of these categories (exhaustiveness).*

3.9 Verticality and Absolute Orientation

We assume that there is a human sensor asserting that a step is aligned with gravity, as illustrated by the *well* example in Sect. 3.2. This primitive is called $VertAln$. Assuming collinearity and parallelism for this primitive would be an oversimplification, because *gravity lines through the earth's body are not straight lines* (compare Fig. 5).

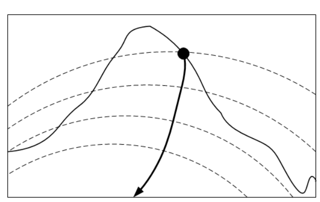

Fig. 5. A curved gravity line and equipotential surfaces.

We assume that *VertAln* describes a symmetric and transitive step, and that it always exists for arbitrary loci of attention:

Axiom10 ***(ubiquity)*** $\forall^{Locus}x.\exists^{Locus}z.VertAln(x,z)$
Axiom11 ***(symmetry and transitivity)*** $VertAln(x,y) \rightarrow Step(x,y) \wedge VertAln(y,x) \wedge \forall^{Locus}z.(VertAln(x,z) \rightarrow VertAln(y,z))$

4 Defining Water Depth from Direct Perception of a Water Body

It remains to show that the Gibson environment is applicable to our water depth scenario, such that all geographic categories, including the water depth quality, are definable in it.

4.1 Deriving Lengths from the Meaningful Environment

We claimed that all involved categories can be defined from the meaningful environment. What is still missing is a definition for a symbol space of lengths. The domain of lengths L is not part of the environment and is assumed to be an *abstract extensional system* in the sense of [26]. It has two binary relations "smaller than or equal" and "sum" that satisfy Suppes' extensional axioms, and is therefore on a *ratio scale* with one degree of freedom.

A length function $Length$ can be derived as a homomorphism from the set of steps of the meaningful environment into the length space L. It is clear that this mapping is conventional and must itself rely on a semantic datum. We therefore first have to fix a *unit step Step(0,u),* for example the two ends of the platinum bar called *"Mètre des Archives"*. We call one end of this bar 0 and the other one u:

Axiom12 ***(non-trivial unit step)*** $\forall x,y.(x=0 \wedge y=u) \rightarrow (Step(x,y) \wedge x \neq y)$

Second, we map the quaternary symbol $\leq$ on loci to the "smaller than or equal to" symbol on the length space. Third, we map the following definable summation symbol on loci to the summation symbol of the length quality space:

Definition 16. ***(sum of lengths)*** $cx \oplus ez =_L ky \leftrightarrow \exists x',y'.(0x' =_L cx) \wedge (0y' =_L ky) \wedge (x'y' =_L ez) \wedge (OnL(0,x',y') \vee OnL(x',0,y') \vee OnL(0,y',x'))$

A sum of the lengths of two steps is a third step having the expected length. We fourth map all steps with the same length as *Step(0,u)* to the symbol "1", and all other steps *homomorphically* to a number symbol: All steps with equal length are mapped to one and only one number symbol such that the truth of all sentences about $\leq$ and $\oplus$ is preserved.

For convenience, we write $\forall^L$ for a quantifier over lengths in L and use the symbols $=_L, \oplus, \leq$, defined for steps, analogously on lengths. Once a length space for steps is established, we can define a *chain length of a path* recursively as the sum of lengths of any chain of steps on it:

Definition 17. ***(chain length)*** $\forall^L k.ChainLength(p) =_L k \leftrightarrow \exists^{Locus} x, y.EndsOfPath(x, y, p) \wedge (k =_L Length(x, y) \vee (\exists z, p', p''.(p = p' + p'') \wedge (k =_L ChainLength(x, z, p') \oplus ChainLength(z, y, p''))))$

The *length of an arbitrarily shaped path* is then its maximal chain length. Because - in our current infinite theory - steps are assumed to be dense, this must be an infinitesimal approximation:

Definition 18. ***(length of a path)*** $\forall^L k.Length(p) =_L k \leftrightarrow \forall k'.ChainLength(p) = k' \wedge k \leq k' \rightarrow k =_L k'$

4.2 Water Depth

The ratio scaled meter water depth space needs a semantic datum to fix its interpretation, because there is no direct sensor available for it. It is a quality that has to be constructed from others. Informally, in the meta-data of a database, we could say that a water depth of a river is the vertical distance measured between a point on the water surface and the river bed. We now can define these notions from the meaningful environment:

Let us start by defining a diameter of a medium in the meaningful environment: A *diameter* of a medium r is any medium path connecting two parts of its surface, such that *the path is contained in r*:

Definition 19. ***(diameter)*** $Diameter(p, r) \leftrightarrow Medium(r) \wedge P(p, r) \wedge SurfaceOf(s, r) \wedge (\exists^{Locus} x, y, u.EndsOfPath(x, y, p) \wedge P(x + y, s) \wedge x \neq y \wedge P(u, p) \wedge \neg P(u, s))$

Note that what normally would be considered as the *water surface* is a part of our medium surface. We define a *depth of a medium* as the length of a diameter path whose steps are vertically aligned:

Definition 20. ***(depth)*** $\forall^L k.Depth(k, p, r) \leftrightarrow Diameter(p, r) \wedge k =_L Length(p) \wedge (\forall^{Locus} z, u.P(z, p) \wedge P(u, p) \rightarrow VertAln(z, u))$

We can now state that a water depth is a depth of a water body connecting the medium air with the ground. There is an infinite number of water depths for a water body (compare [19]):

Definition 21. ***(water depth)*** $\forall^L k.Waterdepth(k, p, r) \leftrightarrow Depth(k, p, r) \wedge Water(r) \wedge (\exists^{Locus} x, y.EndsOfPath(x, y, p) \wedge (\exists^{Locus} a, b.Touching(x, a) \wedge Touching(y, b) \wedge AirC(a, a) \wedge BodyC(b, b)))$

Sticks and sonars both can be considered as realizations of such a virtual water depth path.

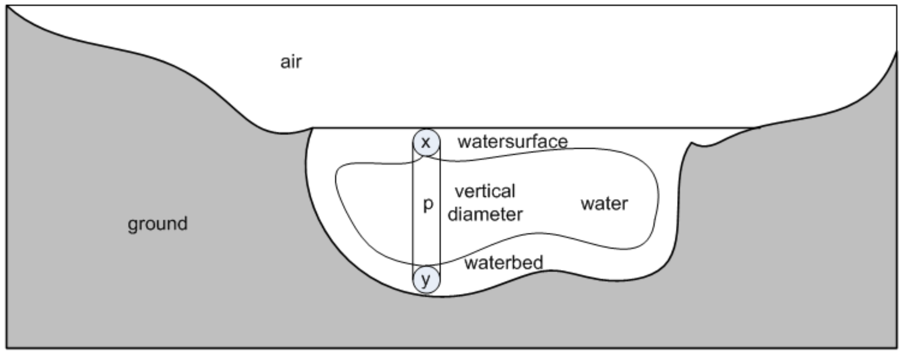

Fig. 6. Water depth is the length of a vertical diameter path in a water body.

5 Conclusion and Future Work

Our approach resolves semantic heterogeneity of basic symbols in an ontology by introducing semantic datums in the form of an observation procedure. Water depth is an example of a basic symbol in a navigation ontology in need of grounding. We show that Gibson's meaningful environment is a sufficient basis to establish such semantic datums. For this, we introduced a formal theory of Gibson's *meaningful environment*, supplying observable primitives for our theory. We show how to ground water depth in our theory by asserting formal characteristics of observable primitives and defining water depth in terms of these.

We take the view that sensors (human or technical) are implementations of semantic datums that reproducibly interpret observable primitives into observable real world structures. Technical sensors are based on human sensors, and human sensors detect bodies and their movements. Natural language semantics is grounded in *bodily experience* and *scanning,* that is the imaginative movement of virtual bodies in a perceivable environment. This is plausible because humans can always unambiguously refer to bodies and their parts by ostension. We claim that Gibson's meaningful environment is fully equipped with body related sensors, including sensors for steps, equal length, equidirection, verticality and various sensors for simple affordance primitives that can be used to individuate media, surfaces and substances.

The presented theory of the meaningful environment is a first sketch of our ideas. As outlined in Sect. 3.5, the theory could be made more appropriate to sensors by making it *finite* and *discrete*, so that we admit a resolution for lengths, along the lines of thought in [24] and [25]. Furthermore, it is still an open question which ontologies are amenable to our method. The general applicability to geospatial concepts needs to be demonstrated with additional case studies. We are currently working on flow velocity and street network categories.

Even though the proposed observation primitives are bound to have cognitive interpretations, it is important to note that they have low indeterminacy. Observable properties on this basic body level do not seem to have a graded structure, too. For example, there is no graded truth value for the sentence "this is the wall of this room" for blind men being in that room, as there will be perfect agreement on this sentence. Any theory about semantic grounding must primarily be able to explain how humans actually accomplish inter-subjective measurement and observation of surface qualities, despite all the cognitive and linguistic ambiguities involved.

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